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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

■ : explanation
∵ is 'because'
∴ is 'therefore'

Pearson Edexcel International Advanced Level

Thursday 9 May 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level

Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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1. Find

$$\int \left(10x^4 - \frac{3}{2x^3} - 7\right) dx$$

giving each term in simplest form.

(3)

① rewrite in easier form for integration

$10x^4 - \frac{3}{2x^3} - 7 = 10x^4 - \frac{3}{2}x^{-3} - 7x^0$

$\because x^0 = 1$

indicies rule: $\frac{a}{cx^b} = \frac{a}{c}x^{-b}$

② Integrate

$\int 10x^4 - \frac{3}{2}x^{-3} - 7x^0 \, dx = \left[\left(\frac{10}{4+1}x^{4+1}\right) + \left(\frac{-3/2}{-3+1}x^{-3+1}\right) + \left(\frac{-7}{0+1}x^{0+1}\right)\right]$

$= 2x^5 + \frac{3}{4}x^{-2} - 7x^1 + C$

DON'T FORGET or will lose a mark

$\therefore 2x^5 + \frac{3}{4}x^{-2} - 7x + C$

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Question 1 continued

(Total for Question 1 is 3 marks)

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2. (i) Given that $m = 2^n$, express each of the following in simplest form in terms of m.

(a) 2^{n+3}

(1)

(b) 16^{3n}

(2)

(ii) **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Solve the equation

$$x\sqrt{3} - 3 = x + \sqrt{3}$$

giving your answer in the form $p + q\sqrt{3}$ where p and q are integers.

(3)

2)i)a) $2^{n+3} = 2^n \times 2^3 = 2^n \times 8$ ① indicies rule : $a^{b+c} = a^b \times a^c$

Let $m = 2^n$

$2^n \times 8 = m \times 8$

$\therefore 2^{n+3} = 8m$

b) $16^{3n} = (16^n)^3 = ((2^4)^n)^3 = ((2^n)^4)^3 = (2^n)^{4\times 3} = (2^n)^{12}$ ②

$3\times n$

② $a^{bc} = (a^b)^c = (a^c)^b$

Let $m = 2^n$

$(2^n)^{12} = (m)^{12}$

$\therefore 16^{3n} = m^{12}$

ii) $x\sqrt{3} - 3 = x + \sqrt{3}$

$+3$ $\quad x\sqrt{3} = x + \sqrt{3} + 3$ $\quad +3$

$-x$ $\quad x\sqrt{3} - x = \sqrt{3} + 3$ $\quad -x$

$x\sqrt{3} - x = 3 + \sqrt{3}$

$x(\sqrt{3} - 1) = 3 + \sqrt{3}$

$\div(\sqrt{3}-1)$ $\quad x = \dfrac{3+\sqrt{3}}{\sqrt{3}-1}$ $\quad \div(\sqrt{3}-1)$

rationalising the denominator :

$x = \dfrac{3+\sqrt{3}}{\sqrt{3}-1} \times \dfrac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = \dfrac{(3+\sqrt{3})(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \dfrac{3\sqrt{3}+3+3+\sqrt{3}}{3+\sqrt{3}-\sqrt{3}-1} = \dfrac{6+4\sqrt{3}}{2}$

$\therefore x = 3 + 2\sqrt{3}$ $\qquad p = 3 \quad q = 2$

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Question 2 continued

(Total for Question 2 is 6 marks)

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3.

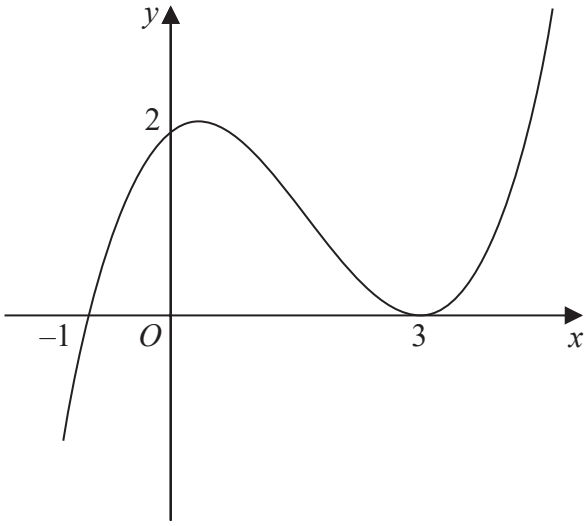


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$.

The curve passes through the points (–1, 0) and (0, 2) and touches the x-axis at the point (3, 0).

On separate diagrams, sketch the curve with equation

(a) $y = f(x + 3)$

(3)

(b) $y = f(-3x)$

(3)

On each diagram, show clearly the coordinates of all the points where the curve cuts or touches the coordinate axes.

a) Working out : $f(x+3)$ Translation $\begin{pmatrix}-3\\0\end{pmatrix}$, move left 3 units. Inside $f(x)$ brackets ∴ only x-coordinates affected & we do inverse of brackets (-3 instead of +3)

$(-1-3, 0)$ & $(0-3, 2)$ & $(3-3, 0)$

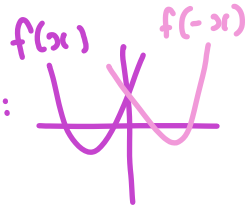
$(-4, 0)$ $(-3, 2)$ $(0, 0)$

b) Working out : $y = f(-3x)$ ① horizontal squash by 3 units. Inside $f(x)$ brackets so only x is affected & inverse is done ($\times\frac{1}{3}$ instead of $\times 3$)

$(-1\times\frac{1}{3}, 0)$ & $(0\times\frac{1}{3}, 2)$ & $(3\times\frac{1}{3}, 0)$

$(-\frac{1}{3}, 0)$ $(0, 2)$ $(1, 0)$

② Transformation reflected vertically by y-axis :



$(-\frac{1}{3}, 0)$ $(0, 2)$ $(1, 0)$

↓ ↓ ↓

$(\frac{1}{3}, 0)$ $(0, 2)$ $(-1, 0)$

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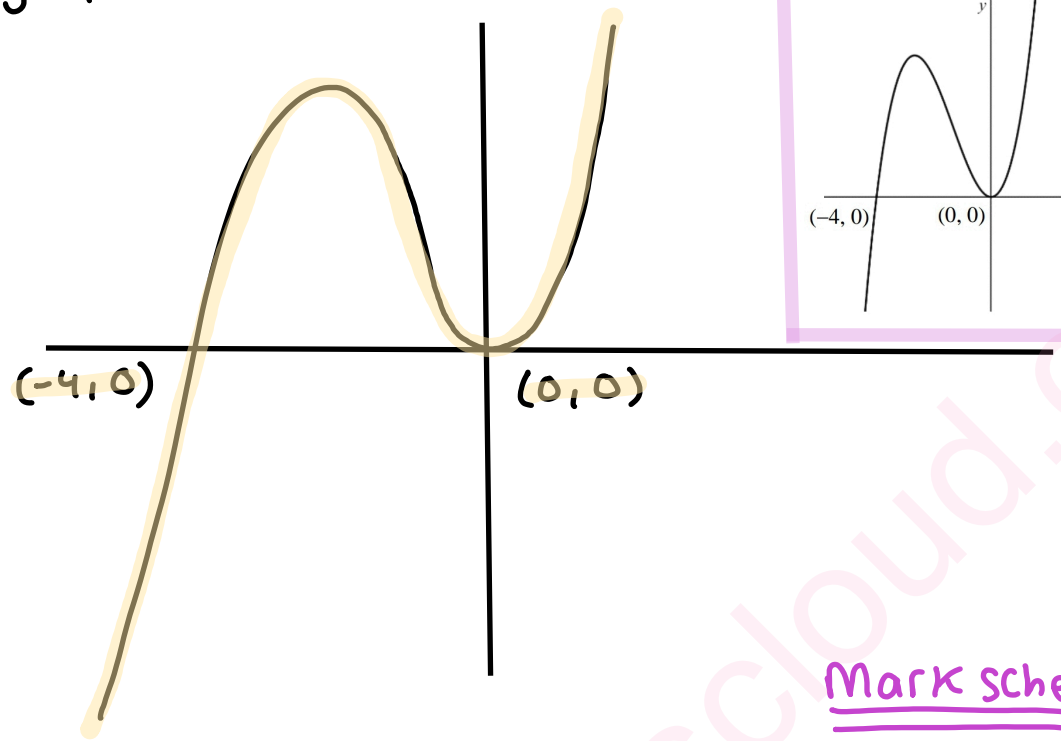
Question 3 continued

a) $y = f(x+3)$



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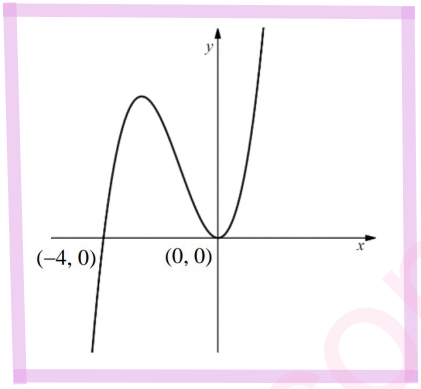
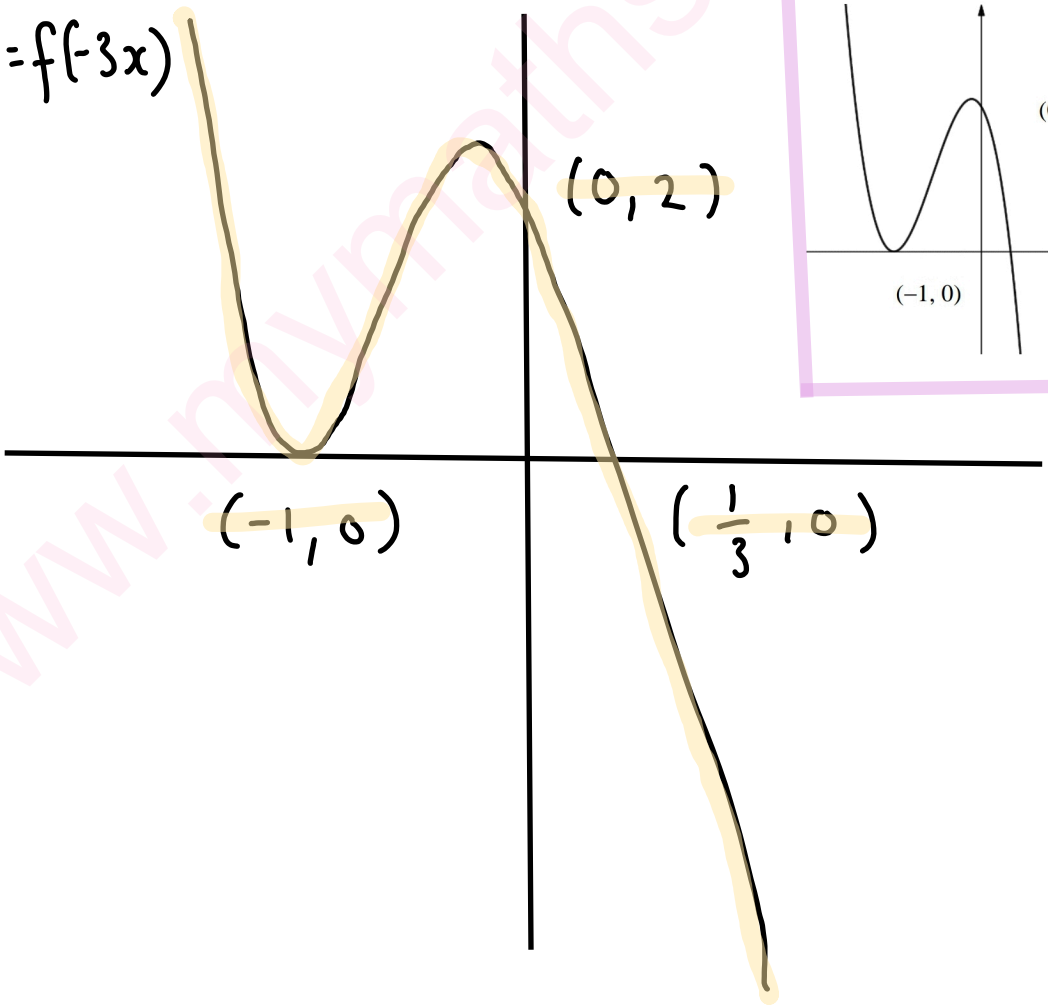


b) $y = f(-3x)$



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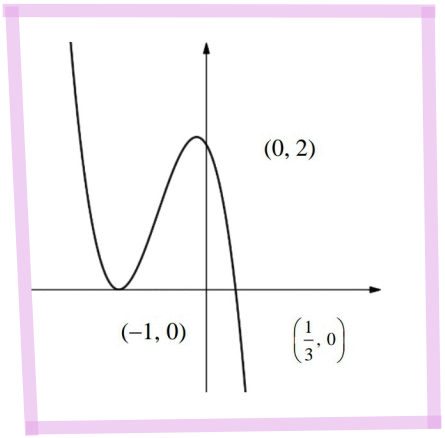

(Total for Question 3 is 6 marks)

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4. The curve C_1 has equation

$$y = x^2 + kx - 9$$

and the curve C_2 has equation

$$y = -3x^2 - 5x + k$$

where k is a constant.

Given that C_1 and C_2 meet at a single point P

(a) show that

$$k^2 + 26k + 169 = 0$$

(3)

(b) Hence find the coordinates of P

(3)

a) ① C_1 & C_2 meet at P ∴ equate C_1 & C_2

$$C_1 = C_2$$

$$x^2 + kx - 9 = -3x^2 - 5x + k$$

$+3x^2$ $\quad 4x^2 + kx - 9 = -5x + k$ $\quad +3x^2$

$+5x$ $\quad 4x^2 + kx + 5x - 9 = k$ $\quad +5x$

$-k$ $\quad 4x^2 + kx + 5x - 9 - k = 0$ $\quad -k$

$$4x^2 + (k+5)x + (-9-k) = 0$$

② Meet at only one point ∴ only one real root so use discriminant formula $b^2 - 2ac = 0$.

One real root means that when the equation of the graph is $ax^2 + bx + c = 0$

discriminant is $b^2 - 4ac = 0$

$$\underset{a}{4}x^2 + \underset{b}{(k+5)}x + \underset{c}{(-9-k)} = 0$$

$$b^2 - 4ac = 0$$

$$(k+5)^2 - 4(4)(-9-k) = 0$$

$$(k^2 + 10k + 25) - (-144 - 16k) = 0$$

$$k^2 + 10k + 25 + 144 + 16k = 0$$

$$k^2 + 26k + 169 = 0$$

$$\therefore k^2 + 26k + 169 = 0$$

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Question 4 continued

b) ① Solve for K.

$K^2 + 26K + 169 = 0$

factorise : $(K+13)(K+13) = 0$ $\quad \therefore (K+13)^2 = 0$

Solve : $K + 13 = 0$

$\therefore K = -13$

② Substitute $K = -13$ into equation $C_1 = C_2$: $4x^2 + (K+5)x + (-9-K) = 0$

$4x^2 + (-13+5)x + (-9-(-13)) = 0$

$4x^2 - 8x + 4 = 0$

$\div 4$ $\quad x^2 - 2x + 1 = 0$ $\quad \div 4$

② Solve for x.

factorise : $(x-1)(x-1) = 0$ $\quad \therefore (x-1)^2 = 0$

Solve : $x - 1 = 0$

$\therefore x = 1$

② Substitute $x = 1$ into equation C_1 or C_2 to find y-coordinate

C_1 : $y = x^2 + Kx - 9$

$y = x^2 - 13x - 9$

$y = (1)^2 - 13(1) - 9 = 1 - 13 - 9 = -21$

$\therefore P(1, -21)$

(Total for Question 4 is 6 marks)

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5.

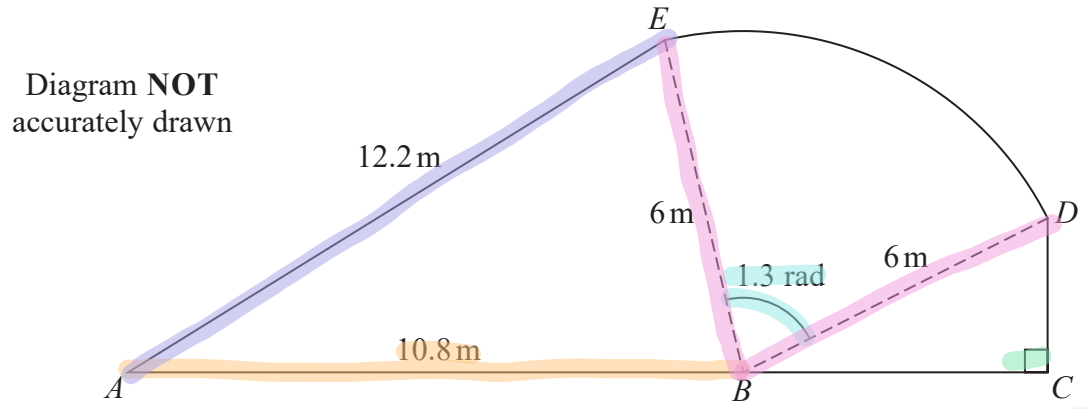


Figure 2

Figure 2 shows the plan view of a garden.

The shape of the garden *ABCDEA* consists of a triangle *ABE* and a right-angled triangle *BCD* joined to a sector *BDE* of a circle with radius 6 m and centre *B*.

The points *A*, *B* and *C* lie on a straight line with $AB = 10.8$ m

Angle $BCD = \frac{\pi}{2}$ radians, angle $EBD = 1.3$ radians and $AE = 12.2$ m

↳ units

(a) Find the area of the sector *BDE*, giving your answer in m^2

(2)

(b) Find the size of angle *ABE*, giving your answer in radians to 2 decimal places.

(2)

(c) Find the area of the garden, giving your answer in m^2 to 3 significant figures.

(3)

a) Area of sector : $A = \frac{1}{2} r^2 \theta$

$r = BE = BD = 6m$

$\theta = \angle EBD = 1.3$ rad

$\therefore A = \frac{1}{2} \times 6^2 \times 1.3 = 23.4$

$\therefore$ Area $= 23.4\ m^2$

b) find $\angle ABE$ using Cosine rule

Pure Mathematics P1

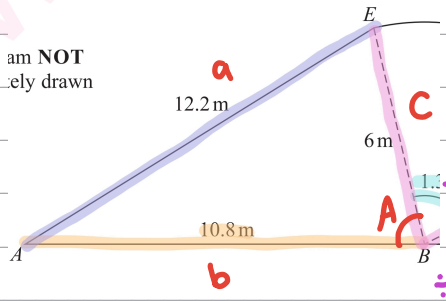
Mensuration

Surface area of sphere $= 4\pi r^2$

Area of curved surface of cone $= \pi r \times$ slant height

Cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$



$(12.2)^2 = (10.8)^2 + (6)^2 - 2(10.8)(6)\cos\angle ABE$

$148.84 = 152.64 - 129.6\cos\angle ABE$ -152.64

$-3.8 = -129.6\cos\angle ABE$ $\div -129.6$

$0.02932\ldots = \cos\angle ABE$

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Question 5 continued

reverse cosine

$0.02932\ldots = \cos\angle ABE$

$\cos^{-1}(0.02932\ldots) = \angle ABE$

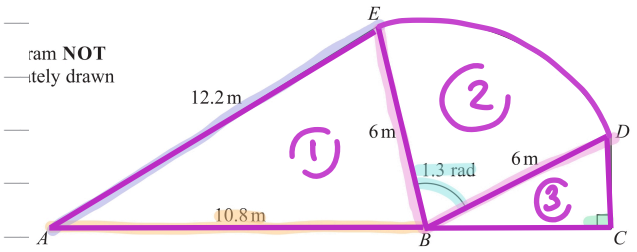
reverse cosine

$1.54147\ldots = \angle ABE$

$\therefore \angle ABE = 1.54$ rad (2 dp)

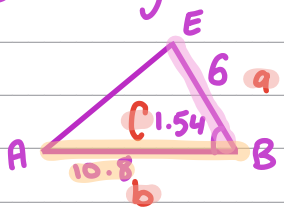
c) Split garden into 3 Shapes.

Area = ① + ② + ③



① Triangle AEB

Area of a triangle: $A = \frac{1}{2}ab\sin C$



$a = EB = 6$

$b = AB = 10.8$

$C = \angle ABE = 1.54$

from part (b)

$A = \frac{1}{2} \times 6 \times 10.8 \times \sin 1.54$

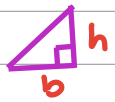
$= 32.3846\ldots \approx 32.38\ m^2$ ①

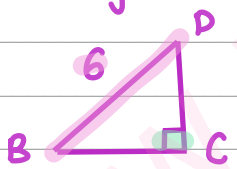
② Sector BDE

Area found in part (a). $A = 23.4\ m^2$ ②

③ Triangle BCD

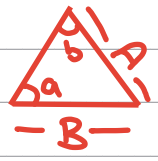
Area of a triangle: $A = \frac{1}{2}bh$





To find length DC, use Sine rule

$$\frac{\sin a}{A} = \frac{\sin b}{B}$$



$$\frac{\sin\angle DCB}{DB} = \frac{\sin\angle DBC}{DC} \rightarrow \frac{\sin\frac{\pi}{2}}{6} = \frac{\sin 0.302}{DC}$$

$\angle DBC = \pi - \angle ABE - \angle EBD = \pi - 1.54 - 1.3 = 0.30159\ldots \approx 0.302$

angle of a straight line is π rad (180°)

$$\frac{1}{6} = \frac{\sin 0.302}{DC}$$

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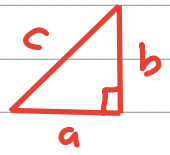
Question 5 continued

$DC = 6\sin 0.302 = 1.7845\ldots$
$\approx 1.78\text{ m}$

To find length BC, use Pythagoras Theorem



$a^2 + b^2 = c^2$

$BC^2 + DC^2 = BD^2$

$BC^2 + 1.78^2 = 6^2$

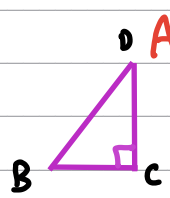
$-(1.78^2)$ $\quad$ $-(1.78)^2$

$BC^2 = 6^2 - 1.78^2$

square root $\quad$ square root

$BC = \sqrt{6^2 - 1.78^2}$

$BC = 5.72988\ldots \approx 5.73\text{m}$



$A = \frac{1}{2} \times 1.78 \times 5.73 = 5.0997\text{m}^2$ (3)

(4) Total Area:

$32.38 + 23.4 + 5.0997 = 60.8797$

$\therefore$ Area $= 60.9\text{m}^2$ (3 sig fig)

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Question 5 continued

(Total for Question 5 is 7 marks)

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6.

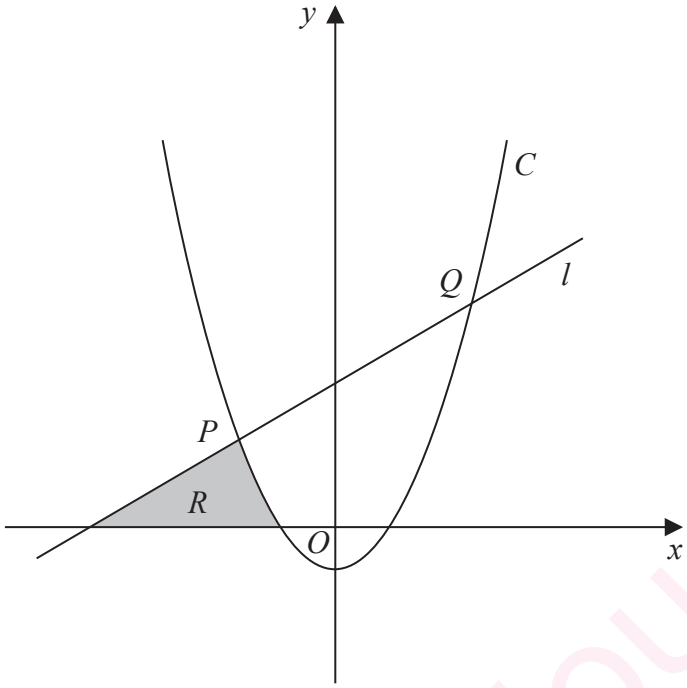

Figure 3

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 3 shows

- the line l with equation $y - 5x = 75$ $\xrightarrow{\text{rearrange}}$ $y = 5x + 75$
- the curve C with equation $y = 2x^2 + x - 21$

The line l intersects the curve C at the points P and Q, as shown in Figure 3.

(a) Find, using algebra, the coordinates of P and the coordinates of Q.

(4)

The region R, shown shaded in Figure 3, is bounded by C, l and the x-axis.

(b) Use inequalities to define the region R.

(3)

a) ① equate l & C

$$L = C$$

$$5x + 75 = 2x^2 + x - 21$$

$-5x$

$$75 = 2x^2 - 4x - 21$$

$-5x$

-75

$$0 = 2x^2 - 4x - 96$$

-75

$$2x^2 - 4x - 96 = 0$$

$\div 2$

$$x^2 - 2x - 48 = 0$$

$\div 2$

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Question 6 continued

② Solve for x.

factorise : $(x-8)(x+6)=0$

Solve : • $x-8=0$ • $x+6=0$

$\therefore x=8$ $\therefore x=-6$

$x>0$ $\therefore$ this is x-coordinate of Q

$x<0$ $\therefore$ this is x-coordinate of P.

③ To find y-values substitute $x=8$ & $x=-6$ into equation l or C.

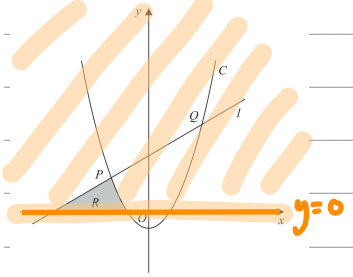
l : $y=5x+75$

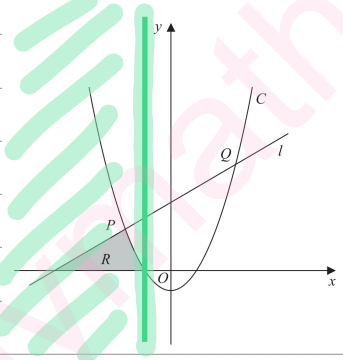
Y-coordinate of Q : $y=5(8)+75=115$

Y-coordinate of P : $y=5(-6)+75=45$

$\therefore P(-6,45)$ & $Q(8,115)$

b) 4 inequalities





① $y \geq 0$ *

② $x \leq -3.5$ *

$\leq$ not $<$ $\because$ line is Solid, not dashed

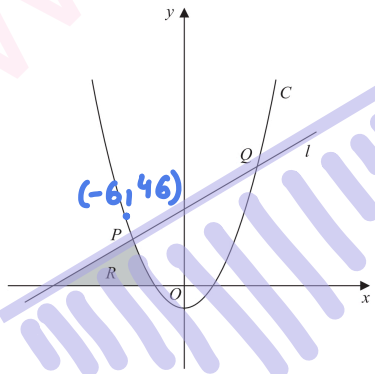
[solid dashed]

negative x-value for C when $y=0$

$0=2x^2+x-21$

FACTORISE : $(2x+7)(x-3)=0$ $2x+7=0 \Rightarrow x=-\frac{7}{2}=-3.5$

③



$y-5x=75$

To find inequality, select point OUTSIDE valid region & substitute into equation & make a FALSE inequality.

Point chosen is $(-6,46)$

$46-5(-6)=75$

$76=75$

Make inequality FALSE: $76 \leq 75$

$\therefore y-5x \leq 75$ *

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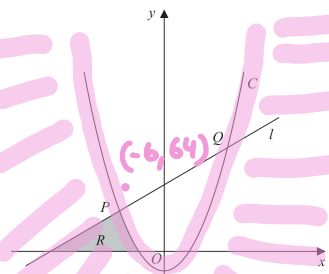
Question 6 continued

4



$y = 2x^2 - x + 21$

To find inequality, select point OUTSIDE valid region & substitute into equation & make a FALSE inequality.

Point chosen is $(-6, 46)$

$46 = 2(-6)^2 + (-6) - 21$

$46 = 45$

Make inequality FALSE: $46 \leq 45$

$\therefore y \leq 2x^2 + x - 21$ *

$\therefore$ Region R is defined by inequalities:

$y \geq 0$, $x \leq -3.5$, $y - 5x \leq 75$, $y \leq 2x^2 + x - 21$

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Question 6 continued

(Total for Question 6 is 7 marks)

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7. The curve C has equation $y = f(x)$ where

$$f(x) = 2x^3 - kx^2 + 14x + 24$$

and k is a constant.

(a) Find, in simplest form,

(i) $f'(x)$

(ii) $f''(x)$

(3)

The curve with equation $y = f'(x)$ intersects the curve with equation $y = f''(x)$ at the points A and B.

Given that the x coordinate of A is 5

(b) find the value of k.

(2)

(c) Hence find the coordinates of B.

(3)

$$f(x) \underset{\text{integrate}}{\overset{\text{differentiate}}{\rightleftarrows}} f'(x) \underset{\text{integrate}}{\overset{\text{differentiate}}{\rightleftarrows}} f''(x)$$

a) i) $f(x) \rightarrow f'(x)$

$\because x^0 = 1$

$f'(x) = 3(2x^{3-1}) + 2(-kx^{2-1}) + 1(14x^{1-1}) + 0(24x^{0-1})$

$= 6x^2 - 2kx + 14$

$\therefore f'(x) = 6x^2 - 2kx + 14$

ii) $f'(x) \rightarrow f''(x)$

$f''(x) = 2(6x^{2-1}) + 1(2kx^{1-1}) + 0(14x^{0-1})$

$= 12x - 2k$

$\therefore f''(x) = 12x - 2k$

b) At point A when $f'(x)$ & $f''(x)$ intersect, $x = 5$.

① equate $f'(x)$ & $f''(x)$

$f'(x) = f''(x)$

$6x^2 - 2kx + 14 = 12x - 2k$

② Substitute $x = 5$ then solve for k.

$6(5)^2 - 2k(5) + 14 = 12(5) - 2k$

$150 - 10k + 14 = 60 - 2k$

$164 - 10k = 60 - 2k$

$+10k$ $\quad 164 = 60 + 8k$ $\quad +10k$

-60 $\quad 104 = 8k$ $\quad -60$

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Question 7 continued

$$104 = 8K$$ $\div 8$

$$13 = K$$ $\div 8$

$$\therefore K = 13$$

c) ① equate $f'(x)$ & $f''(x)$

$$f'(x) = f''(x)$$

$$6x^2 - 2Kx + 14 = 12x - 2K \qquad \boxed{K = 13}$$

$$6x^2 - 2(13)x + 14 = 12x - 2(13)$$

$$6x^2 - 26x + 14 = 12x - 26$$

$-12x$ $\quad 6x^2 - 38x + 14 = -26$ $\quad -12x$

$+26$ $\quad 6x^2 - 38x + 40 = 0$ $\quad +26$

$\div 2$ $\quad 3x^2 - 19x + 20 = 0$ $\quad \div 2$

② Solve for x.

factorise: $(3x-4)(x-5) = 0$

Solve : • $3x - 4 = 0$ $\qquad$ • $x - 5 = 0$

$3x = 4$ $\qquad x = 5$

$\therefore x = \frac{3}{4}$ $\qquad$ ↳ This is x-coordinate of A $\therefore x = 5$

↳ $\therefore$ this is x-coordinate of B

③ Find y-coordinate of B by substituting $x = \frac{4}{3}$ into $f'(x)$ or $f''(x)$

$$f''(x) = 12x - 26$$

$$f''\left(\tfrac{4}{3}\right) = 12\left(\tfrac{4}{3}\right) - 26 = 16 - 26 = -10$$

$$\therefore B\left(\tfrac{4}{3}, -10\right)$$

(Total for Question 7 is 8 marks)

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8. The curve C_1 has equation

$$y = x(4 - x^2)$$

(a) Sketch the graph of C_1 showing the coordinates of any points of intersection with the coordinate axes.

(3)

The curve C_2 has equation $y = \dfrac{A}{x}$ where A is a constant.

(b) Show that the x coordinates of the points of intersection of C_1 and C_2 satisfy the equation

$$x^4 - 4x^2 + A = 0$$

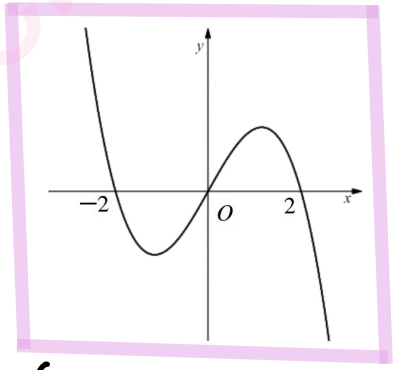
(1)

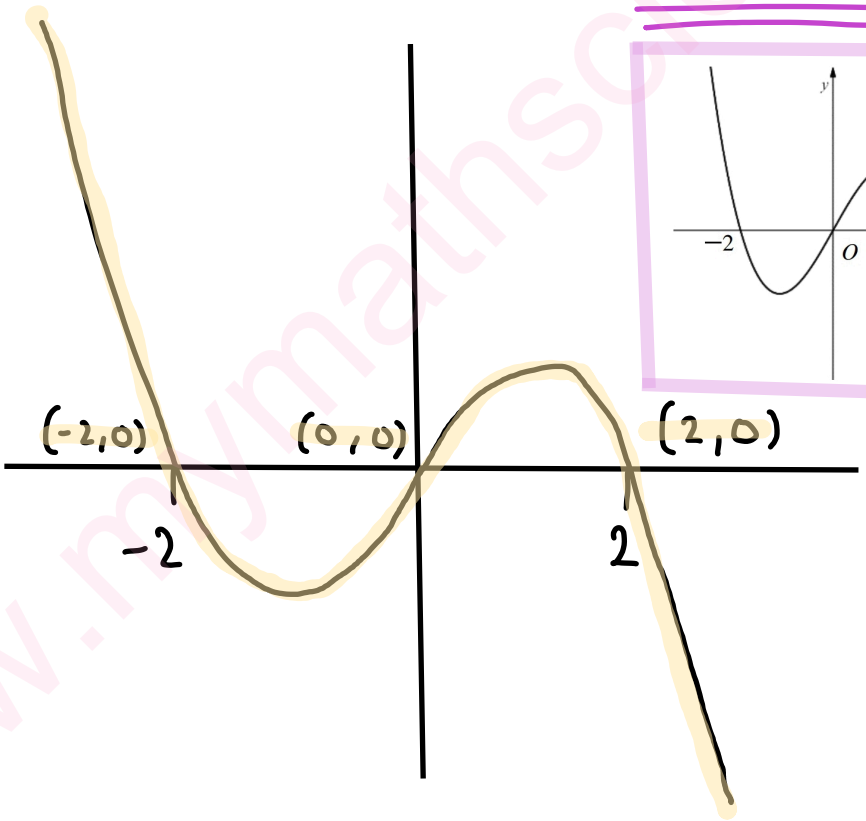
(c) Hence find the range of possible values of A for which C_1 meets C_2 at 4 distinct points.

(3)

a)

Markscheme:





Working out to sketch graph: $x(4-x^2)$ presence of x^3 indicates cubic graph so shape is N or ↳.

$\because$ $x(4-x^2)$ negative cubic graph so shape is ↳.

When $y=0$ $x(4-x^2)=0$ $\therefore$ $x=0$ & $(4-x^2)=0 \Rightarrow x^2=4$

$x = \pm\sqrt{4}$

$\therefore x = \pm 2$

This means graph passes through $(0,0)$ & $(2,0)$ & $(-2,0)$

When $x=0$ $y = 0(4-0)^2 = 0$ y intercept is $(0,0)$

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Question 8 continued

b) ① Since C_1 & C_2 intersect, equate C_1 & C_2

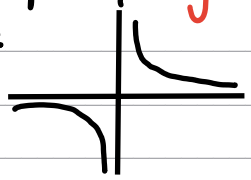
$$C_1 = C_2$$
$$x(4-x^2) = \frac{A}{x}$$

② Simplify

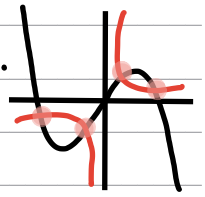
$$4x - x^3 = \frac{A}{x}$$
$\times x$ $\quad 4x^2 - x^4 = A \quad$ $\times x$
$+x^4$ $\quad 4x^2 = x^4 + A \quad$ $+x^4$
$-4x^2$ $\quad 0 = x^4 - 4x^2 + A \quad$ $-4x^2$

$$\therefore x^4 - 4x^2 + A = 0$$

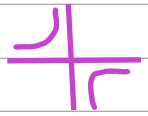
c) ① graph of $y = \frac{A}{x}$ is :



A must be positive so that graph shape so that it intersects 4 times with C_1.



if A was negative, graph would be:



$$\therefore 0 < A$$

② Use discriminant formula $b^2 - 4ac > 0$ for more than 1 real root. four real roots means that when the equation of the graph is $ax^4 + bx^2 + c = 0$

discriminant is $b^2 - 4ac > 0$

$$\underset{a}{1}x^4 \underset{b}{-4}x^2 + \underset{c}{A} = 0$$

$$b^2 - 4ac > 0$$
$$(-4)^2 - 4(1)(A) > 0$$
$$16 - 4A > 0 \longrightarrow 16 > 4A$$
$\div 4$ $\quad 4 > A \quad$ $\div 4$

$$\therefore 0 < A < 4$$

(Total for Question 8 is 7 marks)

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9. Given that

- the point A has coordinates (4, 2)
- the point B has coordinates (15, 7)
- the line l_1 passes through A and B

(a) find an equation for l_1, giving your answer in the form $px + qy + r = 0$ where p, q and r are integers to be found.

(3)

The line l_2 passes through A and is parallel to the x-axis.

The point C lies on l_2 so that the length of BC is $5\sqrt{5}$

(b) Find both possible pairs of coordinates of the point C.

(4)

(c) Hence find the minimum possible area of triangle ABC.

(2)

a) ① find gradient of l_1 using gradient formula $m = \frac{y_1 - y_2}{x_1 - x_2}$

A (4, 2) B (15, 7)

$m = \frac{7-2}{15-4} = \frac{5}{11}$

② Find equation l_1 using line passing through (a, b) & gradient m.

equation: $(y - b) = m(x - a)$

use A(4, 2)

$a = 4$

$b = 2$

$m = \frac{5}{11}$

$(y - 2) = \frac{5}{11}(x - 4)$

③ Write in form $px + qy + r = 0$

$y - 2 = \frac{5}{11}(x - 4)$

$\times 11$ $\quad 11y - 22 = 5(x - 4)$ $\quad \times 11$

$11y - 22 = 5x - 20$

$-11y$ $\quad -22 = 5x - 11y - 20$ $\quad -11y$

$+22$ $\quad 0 = 5x - 11y + 2$ $\quad +22$

$\therefore l_1: 5x - 11y + 2 = 0$ $\quad p = 5 \quad q = -11 \quad r = 2$

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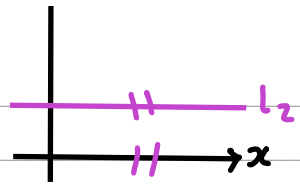
Question 9 continued

b) ① l_2 parallel to x-axis $\therefore$ gradient $= 0$ $\because$



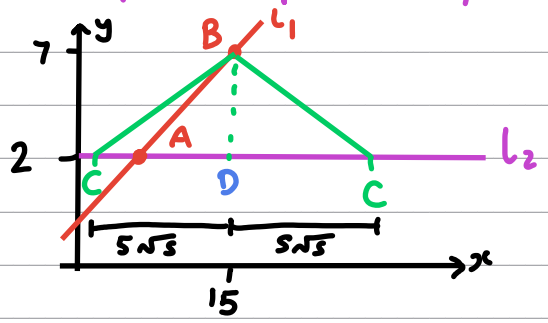
l_2 passes through $A(4,2)$.

$y - 2 = 0(x-4)$

$y - 2 = 0$

$\therefore\ l_2 : y = 2$

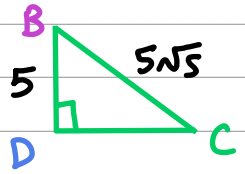
② Visualise 2 possible positions of C.



$A(4,2)$
$B(15,7)$

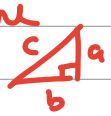
D is $(15,2)$

③ form right angle triangle

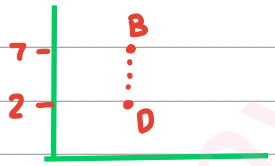


use Pythagoras Theorem

$a^2 + b^2 = c^2$



$BD = 7 - 2 = 5$



$BD^2 + DC^2 = BC^2$

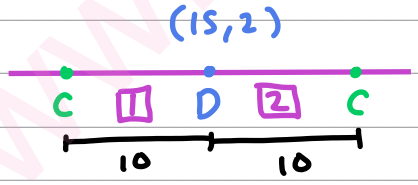
$5^2 + DC^2 = (5\sqrt{5})^2$

$25 + DC^2 = 125$

-25 $\quad DC^2 = 100 \quad$ -25

square root $\quad DC = 10 \quad$ square root

④ Use distance $DC = 10$ to find coordinates of C.



$\boxed{1}$: $(15,2) \to (15-10, 2) \to (5,2)$

$\boxed{2}$: $(15,2) \to (15+10, 2) \to (25,2)$

$\therefore\ C(5,2)$ & $C(25,2)$

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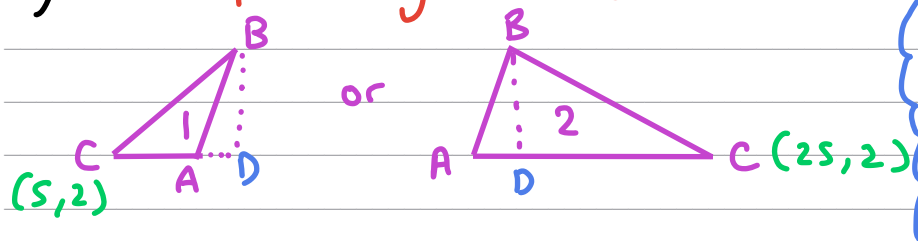
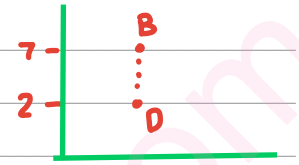
Question 9 continued

c) Area of a triangle : $A = \frac{1}{2}bh$



height is 5 for both triangles ∵

BD = 7 − 2 = 5



A is (4, 2)

in triangle 1, base is 5 − 4 = 1

in triangle 2, base is 25 − 4 = 21

To find minimum area, smaller base should be chosen. ∴ base = 1

$\therefore A = \frac{1}{2} \times 1 \times 5 = \frac{5}{2}$

∴ Area $= \frac{5}{2}$ units^2

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Question 9 continued

(Total for Question 9 is 9 marks)

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10. The curve C has equation $y = f(x)$ where $x > 0$

Given that

- $f'(x) = 6x - \dfrac{(2x-1)(3x+2)}{2\sqrt{x}}$
- the point $P(4, 12)$ lies on C

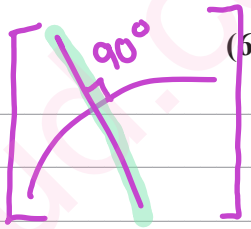
(a) find the equation of the normal to C at P, giving your answer in the form $y = mx + c$ where m and c are integers to be found,

(4)

(b) find $f(x)$, giving each term in simplest form.

(6)

a) normal is Perpendicular to Curve



∴ we find gradient of normal (m_n) using Perpendicular gradient rule $m_{normal} \times m_{curve} = -1$

① find gradient of curve by substituting $P=4$ into the differential $f'(x)$ (gradient function)

$$f'(4) = 6(4) - \frac{(2(4)-1)(3(4)+2)}{2(\sqrt{4})} = 24 - \frac{7 \times 14}{4} = -\frac{1}{2}$$

② find gradient of normal using perpendicular gradient rule

$$m_n \times m_c = -1$$

$$\div -\tfrac{1}{2} \quad m_n \times -\tfrac{1}{2} = -1 \quad \div -\tfrac{1}{2}$$

$$m_n = 2$$

③ find equation of normal using line passing through (a, b) gradient m

equation: $(y - b) = m(x - a)$

$a = 4$
$b = 12$
$m = 2$

$$(y - 12) = 2(x - 4)$$

④ rewrite in form $y = mx + c$

$$+12 \quad y - 12 = 2x - 8 \quad +12$$

$$y = 2x + 4$$

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Question 10 continued

$\therefore \quad y = 2x + 4$

b) $f(x) \underset{\text{integrate}}{\overset{\text{differentiate}}{\rightleftarrows}} f'(x)$

① rewrite $f'(x)$ in easier form for integration.

$$f'(x) = 6x - \frac{(2x-1)(3x+2)}{2\sqrt{x}} = 6x - \frac{(6x^2+x-2)}{2x^{1/2}}$$

① indicies rule: $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

$$f'(x) = 6x - \frac{6x^2+x-2}{2x^{1/2}} = 6x - \left(\frac{6x^2}{2x^{1/2}} + \frac{1x}{2x^{1/2}} - \frac{2}{2x^{1/2}}\right)$$

$$= 6x - \left(3\frac{x^2}{x^{1/2}} + \frac{1}{2}\frac{x^1}{x^{1/2}} - \frac{1}{x^{1/2}}\right) = 6x - \left(3x^{2-1/2} + \frac{1}{2}x^{1-1/2} - 1x^{-1/2}\right)$$

② indicies rule: $\frac{a^b}{a^c} = a^{b-c}$ ③ indicies rule: $\frac{a}{x^b} = ax^{-b}$

$$f'(x) = 6x - \left(3x^{3/2} + \frac{1}{2}x^{1/2} - x^{-1/2}\right) = 6x^1 - 3x^{3/2} - \frac{1}{2}x^{1/2} + 1x^{-1/2}$$

② Integrate

$$f(x) = \int f'(x)\,dx = \left[\left(\frac{6}{1+1}x^{1+1}\right) + \left(\frac{-3}{\frac{3}{2}+1}x^{3/2+1}\right) + \left(\frac{-1/2}{1/2+1}x^{1/2+1}\right) + \left(\frac{1}{-1/2+1}x^{-1/2+1}\right)\right]$$

$$= 3x^2 - \frac{6}{5}x^{5/2} - \frac{1}{3}x^{3/2} + 2x^{1/2} + C$$

③ find value of $+C$ by substituting $P(4, 12)$ so that $f(4) = 12$

$$f(4) = 3(4)^2 - \frac{6}{5}(4)^{5/2} - \frac{1}{3}(4)^{3/2} + 2(4)^{1/2} + C = 12$$

$$48 - \frac{192}{5} - \frac{8}{3} + 4 + C = 12$$

$$\frac{164}{15} + C = 12$$

$-\frac{164}{15}$ $\qquad$ $-\frac{164}{15}$

$$C = \frac{16}{15}$$

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Question 10 continued

$\therefore \quad f(x) = 3x^2 - \frac{6}{5}x^{\frac{5}{2}} - \frac{1}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + \frac{16}{15}$

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Question 10 continued

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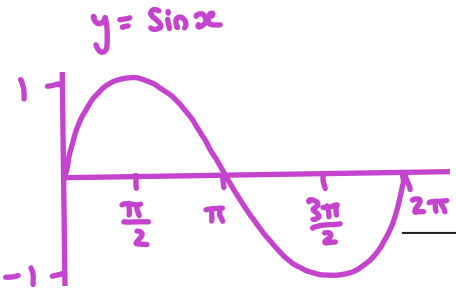
11.



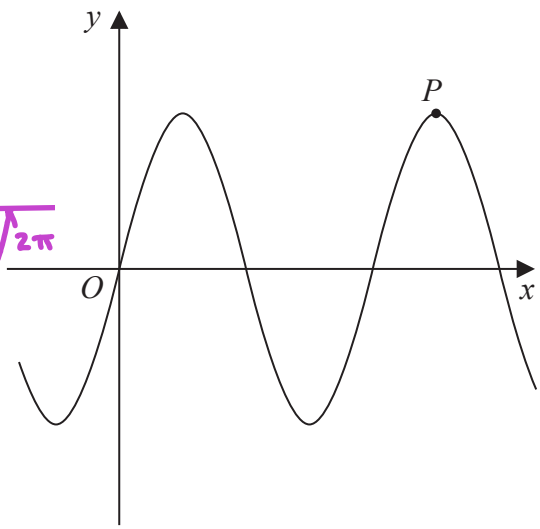


Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 12\sin x$$

where x is measured in radians. units

The point P shown in Figure 4 is a maximum point on C_1

(a) Find the coordinates of P.

(2)

The curve C_2 has equation

$$y = 12\sin x + k$$

where k is a constant.

Given that the **maximum** value of y on C_2 is 3

(b) find the coordinates of the **minimum** point on C_2 which has the **smallest** positive x coordinate.

(2)

The curve C_3 has equation

$$y = 12\sin(x + B)$$

where B is a positive constant.

Given that $\left(\frac{\pi}{4}, A\right)$, where A is a constant, is the **minimum** point on C_3 which has the **smallest** positive x coordinate,

(c) find

(i) the value of A,

(ii) the smallest possible value of B.

(2)

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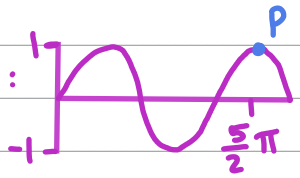
Question 11 continued

a) P is 3rd turning point in $y = 12\sin x$.

3rd turning point of $y = \sin x$ is:



$P(\frac{5}{2}\pi, 1)$

if $f(x) = \sin x$, $12f(x) = 12\sin x$ ∴ transformation is vertical stretch by 12.
12 is outside f(x) brackets ∴ only y-coordinates affected ∴ for P:

$(\frac{5}{2}\pi, 1\times 12) \rightarrow (\frac{5}{2}\pi, 12)$

$\therefore P(\frac{5}{2}\pi, 12)$

b) ① find out value of K.
in part (a) we established that maximum y-coordinate of $y = 12\sin x$ is 12.

if $12f(x) = 12\sin x$, then $12f(x) + K = 12\sin x + K$
translation $\binom{0}{K}$, move up/down by K units.

$12 + K = 3$
$-12 \quad K = -9 \quad -12$

$\therefore y = 12\sin x - 9$

② find minimum at smallest x-value.
in $f(x) = \sin x$:

$(\frac{3}{2}\pi, -1)$

in $12f(x) = 12\sin x$: $(\frac{3}{2}\pi, -1\times 12) \longrightarrow (\frac{3}{2}\pi, -12)$

in $12f(x) - 9 = 12\sin x - 9$: $(\frac{3}{2}\pi, -12-9) \longrightarrow (\frac{3}{2}\pi, -21)$

$\therefore (\frac{3}{2}\pi, -21)$

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Question 11 continued

c) i) $\left(\frac{\pi}{4}, A\right)$ A is the minimum y-coordinate of $y = 12\sin(x+B)$. $\Rightarrow 12f(x+B) = 12\sin(x+B)$

Only 12 is outside f(x) brackets so only effect on y-coordinates is vertical stretch by 12. Hence, same minimum y-value as $y = 12\sin x$

$\therefore A = -12$

ii) in $y = \sin x$, smallest minimum point is $\left(\frac{3}{2}\pi, -1\right)$ when $x > 0$

Smallest positive x-coordinate where minimum occurs is $x = \frac{3}{2}\pi$

in $y = 12\sin(x+B)$ Smallest x-coordinate where minimum occurs is $x = \frac{\pi}{4}$, as stated in the question.

$12f(x+B) = 12\sin(x+B)$ as B is positive this means translation of $\begin{pmatrix} -B \\ 0 \end{pmatrix}$, graph moves left by B units. (when translating x-coordinates we do inverse, so $f(x+a)$ means move $-a$ units).

$\therefore \frac{3}{2}\pi - B = \frac{\pi}{4}$

$-\frac{3}{2}\pi \quad -B = -\frac{5}{4}\pi \quad -\frac{3}{2}\pi$

$\div -1 \quad B = \frac{5}{4}\pi \quad \div -1$

$\therefore B = \frac{5}{4}\pi$

(Total for Question 11 is 6 marks)

TOTAL FOR PAPER IS 75 MARKS

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